

# SYMMETRY-FREE PERIODIC ORBITS OF TURBULENT PIPE FLOW

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**Abstract:** Invariant unstable solutions such as unstable periodic orbits (UPOs), in pipe flow more known as (relative) periodic orbits (RPOs) [1, 2, 3, 4] and travelling waves (TWs) [5, 6, 7] have been suggested to act as building blocks of turbulence in basic shear flows. A large number of such invariant solutions have been determined in recent years, yet most observed states typically possess spatial symmetry (e.g. to rotation, reflection or translation) due to artificial symmetry restrictions. In contrast turbulence does not have any of such symmetry in general. Commonly used recurrence methods [3] are unlikely to capture orbits in full space due to their complexity and the short visiting times of turbulent trajectories.

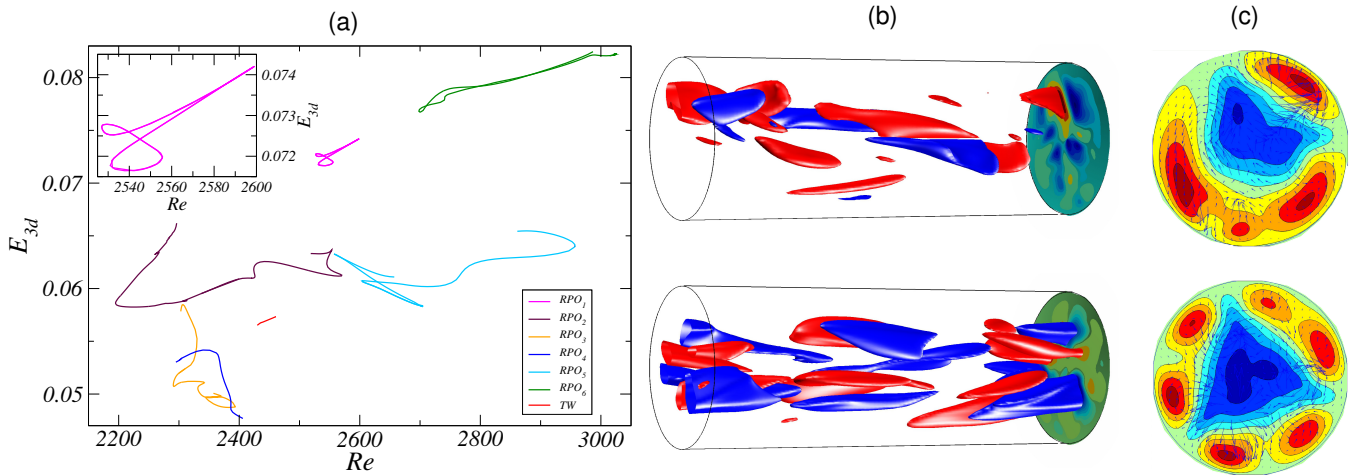


Fig 1: (a) Variation of  $E_{3d}$  with  $Re$  for RPOs (index numbers represent the period time) and TWs as indicated. Flow visualization of (top)  $RPO_{6.511}$  ( $Re = 2550$ ) and (bottom)  $TW$  ( $Re = 2450$ ); Shown are streamwise velocity  $u_z = \pm 0.4U$  (red is positive and blue is negative; flow from left to right) and space-time averaged cross section perpendicular to the pipe axis. The downstream velocity relative to the parabolic laminar profile is shown in color ranging from red (fast) to blue (slow). In-plane velocity components are indicated by vectors.

Nevertheless, looking for periodic modulations instead of full recurrences we have been able to extract asymmetric invariant solutions (Fig. 1), RPOs and TWs, dynamically embedded in turbulence. Compared to other invariant solutions in subspaces the iso-surfaces look less smooth indicating various different length scales within these structures. This is a typical observation in turbulent flows which also show strong fluctuations, e.g. in the internal arrangement of high and low velocity streaks. The complexity of the underlying manifold results in closed curves either in  $Re$  and  $k$  defining parameters with up to four solutions for a single RPO.

**Keywords:** invariant solutions, pipe flow, turbulence, unstable periodic orbits.

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